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Holistic Methods

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INTRODUCTION

The theory of magnetic monopoles makes us reconsider topological assumptions that lie rather close to the foundations of physics, such as the smoothness of time space. The strings Dirac attached to his monopoles are among the earliest of the topological variables of physics, which now include string and bag hadron models, holes and singularities of curved time spaces, and homotopic charges (kinks) of topologically nonlinear field theories. Here I shall outline how far this topological trend has gone in physics, and how far I think it will go. Topological methods are mainly useful in nonlinear problems, and this trend to topology is part of an ongoing delinearization of physical theory resulting from its progressive deepening. Since Newton's First Law, linear theory has described just the asymptotic and uneventful-seeming parts of the physical process. More and more we recognize lately that no part of the world is actually uneventful, and that linearity represents only the coarseness of our resolution of the fine structure of the physical process. The deepening of theory that leads to delinearization and topologization is the increase in our resolution of that fine structure, above all in our search for the origin of "elementary" particles or quanta, and the approach to a holistic physics.

The path to foundations I am following (there are others) can be represented by six steps.

1. "LINEAR" FIELD THEORY

This is the level of Maxwell's equations for the free electromagnetic field, with canonical commutation relations. The existence and the action scale of

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the photon come from the quadratic commutation relation, the sole nonlinear element of this "linear" theory. (Of course, the existence of particles is always associated with some nonlinearity.) But where do electrons (photon sources) come from?

2. NONLINEAR FIELD EQUATIONS

Even before the quantum theory, Mie, Einstein, Weyl, and others tried to make particles out of blobs of field. They believed that nonlinear self-interactions would cause the field to clot. Right they were, though it took the development of electronic digital computers to prove it; these surprisingly particle-like nonlinear field objects are often called solitons today. Are electric charges solitons of the electromagnetic field? This is the most ambitious application of soliton theory, requiring a new set of electromagnetic field equations like Born and Infeld's. Where would the needed modification of the Maxwell field equations come from?

3. NONLINEAR FIELD VARIABLES

There are field theories where that question is less urgent because the field variables are themselves topologically nonlinear, admit no continuous superposition, and have no linear theory.

Objects called *topological solitons*, or *kinks*, or *homotopic charges*, may then arise that are preserved in number not merely by the dynamic evolution but by *any* continuous evolution or homotopy.

The simplest example is Skyrme's equation $\Box\varphi + \sin\varphi = 0$, where the range of the field variable φ is a circle, not a line. Skyrme's solitons are well known.

The most familiar and important example is Einstein's equation $R_{\mu\nu} = 0$, where the range of the field variable $g_{\mu\nu}$ is the set of symmetric tensors of signature 1-3, a quotient space of the group $GL(4, R)$ (by a Lorentz subgroup). Kinks in $g_{\mu\nu}$ were important in initiating the modern work on black holes. (Spherical $g_{\mu\nu}$ kinks are surrounded by trapped surfaces.)

An ambitious example is Pisello's (1978) suggestion that electric charges are indeed homotopic charges, topological solitons, related to photons as Skyrme's kinks are to the small oscillations in Skyrme's field. Pisello's field p has range S^2, the unit sphere in a three-dimensional space, and is related to the electromagnetic field $F_{\mu\nu}$ by

$$F_{\mu\nu} = \epsilon_{\alpha\beta}\partial_\mu p^\alpha \partial_\nu p^\beta$$

where the p^α are local coordinates on S^2 with spherical volume element $(dp^2) = dp^1 dp^2$, and $\epsilon_{\alpha\beta}$ is the 2×2 Levi–Cevita form. Pisello's theory has some remarkable features: the current density j^μ of its homotopic charge is, by a result of Whitehead,

$$j^\mu = \tilde{F}^{\mu\nu} A_\nu \qquad \tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\alpha} F_{\alpha\beta} \epsilon^{\nu\beta}$$

(where A_ν is any vector potential for $F^{\mu\nu}$) and is just the Maxwell current density $\partial_\mu F^{\mu\nu}$ by virtue of the variational principle $\delta \int F^2 d^4x = 0$ for the field $p(x)$, where $F^2 = F^{\mu\nu} F_{\mu\nu}$ as usual.

As Pisello's model illustrates, gauge theories can be found on this level.

4. NONLINEAR GLOBAL TIME SPACE

Einstein's theory of gravity leads one to abandon the topological linearity of time space as well as field space, since the simplest solution, that of Schwarzschild, leads by analytic continuation outside the realm of topologically linear time space. I pass quickly by the distracting beauties of this level to reach the next, pausing only to mention an observation of Sorkin (1977) that is particularly relevant to monopole theory. Sorkin divides all space R^3 by a torus $S^1 \times S^1$ into two parts, inside and outside the torus, and discards the inside. He then removes the toroidal boundary of the outside space by an identification of the point (φ, θ) on $S^1 \times S^1$ with $(\varphi, \pi + \theta)$, creating a nonorientable manifold. If a magnetic field is normally outward at (φ, θ), verify that it is normally outward at $(\varphi, \pi + \theta)$ also, by continuity through the torus. It may thus be outward everywhere, yet divergenceless. Thus a monopole requires no magnetic source density or divergence in such a manifold. Sorkin uses this configuration not to model magnetic monopoles, which are scarce, but electric ones, which are plentiful, making E rather than B an axial field for the purpose.

5. NONLINEAR LOCAL TIME SPACE

Physical computations with nonlinear fields in nonlinear time spaces are so difficult and apt to diverge at small distances that they lead us to question the validity of the time space continuum at small distances. A manifold is a locally linear space. There are no operational grounds for this local linearity. Indeed the Bohr–Rosenfeld theory of measurements at a point requires such nonrealistic test charges as to make local linearity doubtful. The main reason for assuming it has been the absence of practical working alternatives. I have attempted to suspend local linearity too.

This requires a new attitude toward the geometric invariance groups of physics: translation, rotation, etc. The usual translation $x \rightarrow x + a$, for example, is either not observable (if it refers to the whole process) or not an exact invariance (if it refers to the object of the experiment only), since the outcome of an experiment depends on where it is carried out in relation to any actual reference system. This can be put more formally as a principle of asymmetry: *There are no symmetries.* Here is why I posit this principle.

Only those properties of the entire situation are observable that are invariant under the automorphisms of the theory. (I learned *this* principle from J. L. Anderson at our first meeting.) Therefore exact automorphisms change no observable properties, and are unobservable. In accord with the operational thrust of quantum mechanics and relativity, I eliminate these unobservable transformations, and all exact symmetries. I therefore assume there are *no* symmetries (in the basic theory of microsystems, demanding that in the macroscopic limit the usual invariances return as a limit of approximate symmetries).

What is left seems to be a world that is a pattern of elementary discrete quantum processes. In one series of attempts, the world is represented as a checkerboard, with two possible directions of motion at each vertex. Instead of $x^\mu = \int dx^\mu$, I put $x^\mu = \tau \sum \gamma^\mu$, with finite noncommutative "differentials." Here translational invariance is renounced but the isotropy of time space is retained by admitting coherent superpositions of the two directions as well, which therefore transform under $SL(2, C)$, which covers the Lorentz group twice. It is possible to describe a Dirac spin-$\frac{1}{2}$ particle in such a quantum time space, with a mass spectrum proportional to $1/\tau$. This theory violates the principle of asymmetry. The surviving symmetry is that of the two-dimensional spin state space, which I now attack.

6. NONLINEAR STATE SPACE

We come now to what must surely be the last linearity, please, that of the Hilbert space of states. The state space is a way of representing the selection rules $A) \rightarrow)B$ for quantum transitions from (pure) effectors $A)$ to receptors $)B$, telling which are forbidden, which are allowed. These determine the linear space uniquely, but not the metric.[2]

The Minkowskian manifold of relativity and the Hilbert space of quantum theory ("time space" and "truth space") seem closer than usual to each other in this theory. They are both ways of presenting certain dyadic relations, the causal relation (Robb's formulation of relativity) and the allowed-transition relation (a formulation of quantum theory). Furthermore, the two relations they represent are quite similar in their operational meaning. They are both *transfer relations*, expressing the possibility of a flow from an effector to a receptor. In relativity these transmit and receive light signals; in

[2] The *compulsory* transitions $A) \sim)B$ determine the metric.

quantum theory, individual systems. Since the former are aggregates of the latter, relativity's "time space" might be an aggregated form of quantum theory "truth space."

If the repertory of processes A) is discrete, so should be the state space. Continuity may arise only as an approximation, depending on the macroscopic nature of the experimental apparatus. Therefore I do not imagine that the state space description of quantum processes extends down to the basic microscopic level. I assume instead that there is a discrete diagram language for maximally describing whole processes.

This is an unusual mode of description. The usual mode I call *objective*: it attempts to suppress the variables of the subject and deal with the variables of the object only, not the whole process. In objective theories many properties that may actually be relative, such as the continuity of location in time space, are considered absolute because they are common to all sufficiently complex observers. The unusual mode we enter here I call *holistic*: it describes the whole situation, as Bohr said the wave function does, but maximally, unlike the usual wave function, which grievously slights the subject.

For diagrams p, p' describing effector and receptor processes, respectively, I suppose a given *null* or *exactness* relation

$$p \circ p'$$

meaning in the macroscopic limit that the system of p is unaffected by p' and not recorded or counted. The quantum analog is $\langle p'|p\rangle = 0$.

The basic process diagrams p may not be enough language for physics. It is part of the concept of a physical determination (and a measurement) that it apply to a plurality of cases, not just one. Therefore a determination (or a measurement) is not a basic process, but a rule applying to various such processes (representing various objects of determination) producing other such processes (representing the determination being carried out on the object), that is, a kind of process on processes or "metaprocess." Though (basic-level) process diagrams may obey the principle of asymmetry, these higher-level diagrams need not; a dual symmetry between range and domain may appear, represented by arrow reversal. Perhaps this is the seed of time space symmetry.

The main difference between classical and quantum logical structures is this: In classical theories, $p \circ p'$ *holds* for all p' but one, depending on p: what is not forbidden is *compulsory*. In quantum theories, $p \circ p'$ *fails* for almost all p': what is not forbidden is *allowed*. But the set of p' for which $p \circ p'$ holds determines p in both classical and quantum theories. That determinacy survives.

Let me indicate how the usual linear space is to be reconstructed from $p \circ p'$, in two stages: (a) the lattice of subspaces, then (b) the linear space.

(a) A basic question about the relation of the mathematical language of the theory to the physical processes it describes is:

What sets P of diagrams p,

$$P = \{p\}$$

are associated with (usually nonmaximal) physical determinations within the system? Each such diagram is to describe an entire experiment, including a physicist and life-support system or an automatic recording system, in which a certain determination is made. If the determination is before the fact (source, not test; effector, not receptor), I call the set of diagrams a predicate.

P is a predicate (I postulate) if and only if there exists a set $P' = \{p'\}$ of receptor processes (determinations after the fact) such that p is in P if and only if $p \circ p'$ for all p' in P'.

Verify these sets P form a lattice. Make *no* assumptions about $p \circ p'$ for this (Birkhoff, 1948).

In a dual way we define a lattice of sets P' of receptor processes, *copredicates*. The two lattices $\{P\}$, $\{P'\}$ are connected by two dually-defined inclusion-reversing maps $P \to P' = P_0$ and $P' \to P = {}_0P$:

$$P_0 = \{p' \text{ in the relation } p \circ p' \text{ to all } p \text{ in } P\}$$
$${}_0P' = \{p \text{ in the relation } p \circ p' \text{ to all } p' \text{ in } P'\}$$

This pair of lattices and mappings is called here the *Galois Connection* (Birkhoff, 1948) of the relation $p \circ p'$. Some important cases are as follows:

1. In Galois' case, p' is an element of a (number) field, p is an automorphism of that field, and $p \circ p'$ means that p leaves p' fixed.

2. Another case: if an integer-valued "inner product" (p, p') is given for diagrams, the relation $(p, p') = 0$, taken for $p \circ p'$, defines such a Galois connection.

3. For another kind of example, take p' to be a point of some fixed universal algebra (or relational system, for that matter), p to be an endomorphism of that algebra (or system), and $p \circ p'$ to mean that p fixes p'. Verify that this case includes case 2, by taking p' to be an element of the module of dimension n

$$N + \cdots + N = nN$$

over the ring N of the integers.

(b) The step from lattice to linear space is well known. If the lattice is reducible, reduce it. The construction is carried out for the irreducible parts, which are related by superselection rules. If one of these lattices is modular, and admits a *polarity* (= an isomorphism $P \leftrightarrow P^*$ between the lattices of physical effectors and physical receptors, with never $p \circ p^*$), usually not unique, then a linear space $L(n, F)$ exists (defined by its dimension n and number field F) such that the physical lattice is the lattice of subspaces of $L(n, F)$ (assuming finite heights >2 for simplicity).

This construction proves that the selection rules determine the linear space as asserted above.

For example, suppose an effector diagram p can be described by integers, forming a column "vector" (module element) (p^A), and a receptor diagram by a dual integer row (p'_A), such that $p \circ p'$ is equivalent to

$$p^A p'_A = 0$$

Then the linear space resulting from the calculations of (a) and (b) is $L(n, Rat)$, of dimension n over the field of rational numbers. From the linear space over the rational numbers we construct the real numbers by completion, and the complex numbers by applying Stone's theorem to the one-parameter unitary group of approximate automorphisms $U(t)$ representing time translation, defining the operator

$$i = \dot{U}U^{-1}/|\dot{U}U^{-1}|$$

and adjoining i as superselection operator. This may be how quantum complex numbers are born.

The diagram language will already have processual elements, like verbs; for instance, point events a, b, and perhaps also the pair $a \rightarrow b$ representing a flux from a to b. I suppose that the null relation $p \circ p'$ between diagrams, since it too expresses the (im)possibility of a certain process, is not an independent element of structure, but must be stated in the language.

After we construct the linear space of states, we must use it to construct the time space of relativity, perhaps by the methods of Section 5. The end of our exploring will be to return to where we started, macroscopic nature, with a rationale for what seems arbitrary at present: the masses and interactions of the particles, which are not at all elementary.

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REFERENCES

Birkhoff, G. (1948). *Lattice Theory*. American Mathematical Society.

Finkelstein, D. (1978). The delinearization of physics, in *Symposium on the Foundations of Modern Physics, Loma-Koli, Finland, 11–18 August 1977* (to be published).

Pisello, D. (1978). Unified theory with homotopic charge, *International Journal of Theoretical Physics* (to be published).

Sorkin, R. (1977). On the relation between charge and topology, *Journal of Physics A*, **10**, 717.